and values of this quantity may now be calculated. Such results for three different Reynolds numbers are indicated in Fig. 4.

In summary, this note has presented a mean velocity profile expression for turbulent flow which is apparently universal in nature. This expression was obtained by modifying and combining two previously proposed expressions for different regions of the flowfield. Values of the Reynolds stress, and of the eddy diffusivity for momentum, obtained using these expressions were presented for Reynolds numbers from 50,000 to 500,000.

References

[1] Schlichting, H., *Boundary Layer Theory*, McGraw-Hill, New York, 1968, Chaps. 19–20.

[2] Kays, W. M., *Convective Heat and Mass Transfer*, McGraw-Hill, New York, 1966, p. 71.

[3] Longwell, P. A., *Mechanics of Fluid Flow*, McGraw-Hill, New York, 1966, p. 325.

[4] Gow, J. L., *Fully-Developed Turbulent Flow in Smooth and Rough-Walled Pipe*, M. S. thesis, 1969, Mechanical Engineering Dept., Montana State Univ., Bozeman, Mont.

[5] Powe, R. E., *Turbulence Structure in Smooth and Rough Pipes*, Ph.D. thesis, 1970, Aerospace and Mechanical Engineering Dept., Montana State Univ., Bozeman, Mont.

[6] Laufer, J., "The Structure of Turbulence in Fully-Developed Pipe Flow," Rept. 1174, 1954, NACA.

Effects of a Flexible Surface on Surface Shear-Stress Fluctuations beneath a Turbulent Boundary Layer

MICHAEL E. MCCORMICK*, BRUCE JOHNSON†, AND WILLIAM M. LEE‡
U.S. Naval Academy, Annapolis, Md.

Nomenclature

C	= elastic wave velocity
f	= frequency
P	= pressure
Q	= quality factor
$U(y)$	= flow velocity
U_c	= convection velocity
y	= normal coordinate to plate
α	= angle of incidence
β	= damping factor
δ	= boundary-layer thickness
τ	= shear stress
ω_1	= circular fundamental frequency
$\langle\ \rangle$	= mean value

Introduction

THE study of turbulent boundary-layer induced vibrations of plates and membranes is most significant in understanding the production of near field sound generated by submerged bodies. In addition to the self-noise problems, this area of study is relevant to the phenomenon of drag reduction by compliant surfaces.

Although the nature of the pressure fluctuations on a surface beneath a turbulent boundary-layer and the behavior of associated excited surfaces are rather well understood,[1–3] the effect of surface compliance on the time-dependent surface shear stress has not been adequately investigated. The significance of such an investigation is that the shear stress fluctuation beneath a turbulent boundary-layer is a local phenomenon and, therefore, not significantly influenced by radiation.

In the present study, a circular membrane is flush-mounted in the surface of a flat plate which is inclined at a negative angle of attack in a low-turbulence subsonic wind tunnel. The boundary-layer over the plate is turbulent due to the presence of trips mounted well upstream of the membrane. The effect of motions of the membrane on the fluctuating surface shear stress is studied by placing a flush-mounted hot-film just downstream of the membrane. The motions of the center of the membrane are measured using a fiber-optic displacement probe.

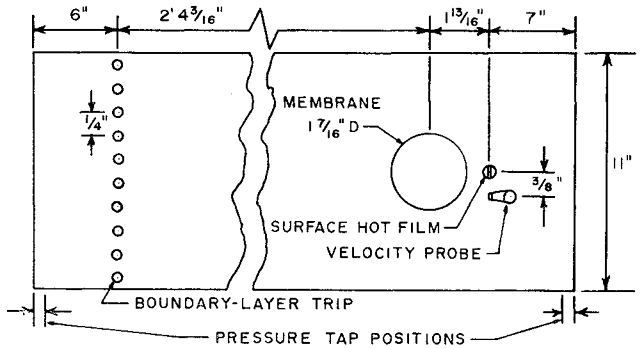

Fig. 1 Schematic diagram of the top of the experimental apparatus.

Experimental Apparatus

An aluminum membrane of 0.00005 in. thickness was stretched over the opening of a $1\frac{7}{16}$ in. i.d. steel tube and mounted in a flat plate as shown in Fig. 1. The plate was mounted in a low-turbulence subsonic wind tunnel at a negative angle of incidence of 3° as shown in Fig. 2. The sides of the plate were lined with foam rubber and pressed against the tunnel side-walls to prevent edge flows. Static pressure taps were located in the tunnel walls above the leading and trailing-edges of the plate to measure the pressure gradient over the plate. Studs $\frac{1}{16}$ in. in height and $\frac{3}{16}$ in. in diameter were situated 6 in. from the leading edge to insure the existence of a turbulent boundary-layer. The centers of the studs were separated by $\frac{1}{4}$ in.

A thermo-systems hot-film velocity probe was mounted $\frac{3}{8}$ in. downstream of the membrane as shown in Figs. 2 and 3. The height of the probe was adjustable so that the velocity profile in the boundary-layer could be measured. The surface shear stress fluctuations were measured using a thermo-systems flush-mounted hot-film probe as shown in Figs. 2 and 3. This probe is $\frac{3}{32}$ in. in diameter with a platinum film mounted on a flat head.

Motions of the center of the membrane were measured using a MTI Fotonic Sensor which is a fiber-optic probe $\frac{1}{8}$ in. in diameter. This probe was mounted $\frac{3}{32}$ in. from the static equilibrium position of the membrane, see Fig. 3. Leads from the probes were enclosed in a fairing beneath the plate.

Signals from the probes were amplified and recorded on a Hewlett-Packard 4-channel tape recorder. A schematic dia-

Table 1 Flow data

Run	P(in. H_2O)	δ(in.)	U(fps)	$\langle U^2\rangle/U_\delta$	$R_\delta(\times 10^{-4})$
1	0.063	0.500	30.5	0.0850	0.747
2	0.137	0.468	58	0.0664	1.250
3	0.195	0.437	68.5	0.0505	1.547
4	0.378	0.406	83.4	0.0414	1.745
5	0.484	0.375	122	0.0284	2.400

* Associate Professor of Ocean Engineering and Chairman, Dept. of Naval Systems Engineering.
† Professor of Ocean Engineering.
‡ Associate Professor of Mechanical Engineering.

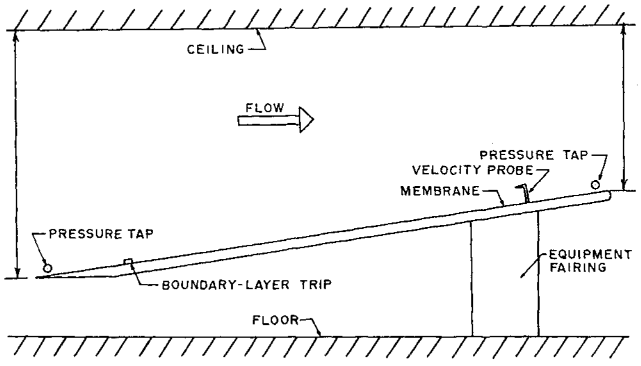

Fig. 2 Diagram of the side view of apparatus.

gram of the data acquisition equipment is shown in Fig. 4. The data on the tapes were subsequently analyzed using a time-data 100 real-time analysis system.

Experiment

To determine the characteristics of the flow, a rigid surface was first inserted into the membrane cavity. With this surface in place, the velocity profile within the boundary-layer was determined for five static pressure conditions. The velocity profiles measured perpendicular to the plate resembled that in the insert of Fig. 5. This type of profile existed because the flow away from the plate was not parallel due to the negative angle of incidence. The boundary-layer thickness is taken to be the point of inflexion in the profile, i.e., where $du/dy = 0$. Data from the five runs are presented in Table 1, and the dimensionless velocity profiles of Runs 1, 3, and 5 are shown in Fig. 5. The turbulence level at $y = \delta$ is indicated by the ratio of the rms velocity and the time averaged velocity.

The rms of the time-dependent shear stress on the surface was measured for all runs. These data are represented in Fig. 6 by the solid lines. The measurements previously mentioned were repeated with the membrane in position. The time-dependent shear stress curves are represented by dashed lines in Fig. 6. Where no dashed lines can be seen, the data from the two conditions coincide. The surface hot-film was not calibrated and, therefore, the data from these measurements are presented in *db* relative to an internal voltage.

Measurements of the rms displacement amplitude of the center of the membrane for a fundamental frequency of 140 Hz are presented as a function of the velocity ratio U_c/C in Fig. 7, where U_c is the convection velocity within the boundary-layer taken to be about $0.8U_\delta$, and where C is the wave velocity in the membrane. These data are also presented in *db* relative to an internal voltage for convenience, although the fiber-optic probe was calibrated. The maximum deflection of the center of the membrane was approximately $\frac{1}{64}$ in.

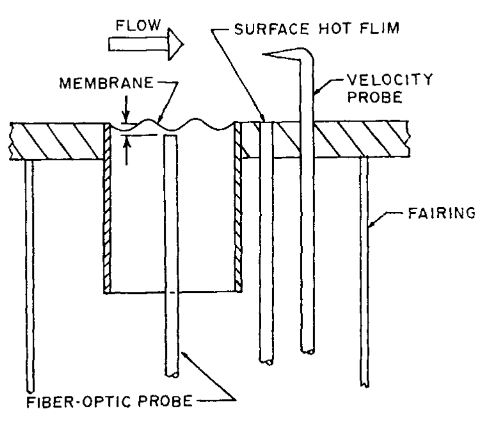

Fig. 3 Diagram of membrane and probes.

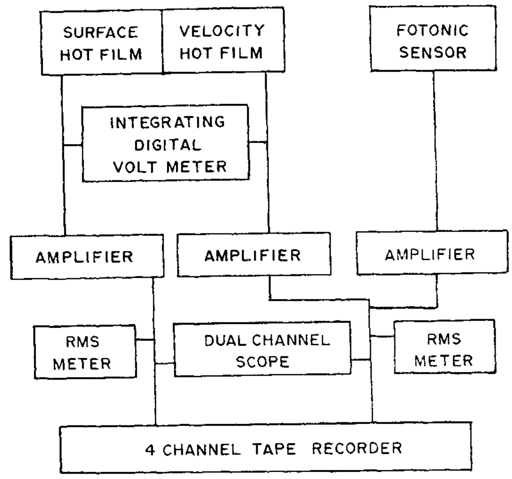

Fig. 4 Schematic diagram of data acquisition equipment.

The damping of the membrane's motions is presented in dimensionless form in Fig. 8 as a function of velocity. This damping was determined from measurements of the half-power bandwidth about the fundamental frequency f_1. The bandwidth, $f_b - f_a$, is determined from the high frequency, f_b, and the low frequency, f_a, 3 *db*-down response values. The dimensionless damping is then

$$\beta_1/\omega_1 = 1/Q = (f_b - f_a)/f_1 \qquad (1)$$

where β_1 is the damping constant, ω_1 is the circular fundamental frequency, and Q is the quality factor.

Results

The results presented in Fig. 6 show that the presence of the membrane has a definite influence on the time-dependent surface shear stress, i.e., causing an increase in the rms values for the first three runs. The velocity dependence of the membrane's effect is similar to that observed in Ref. 4 on the fluctuating surface pressure fluctuations; however, in that referenced study the influence of the compliant surface increases with velocity.

The response of the center of the membrane at the fundamental frequency as a function of flow velocity, shown in Fig. 7, increases to a peak value and then decreases as the flow

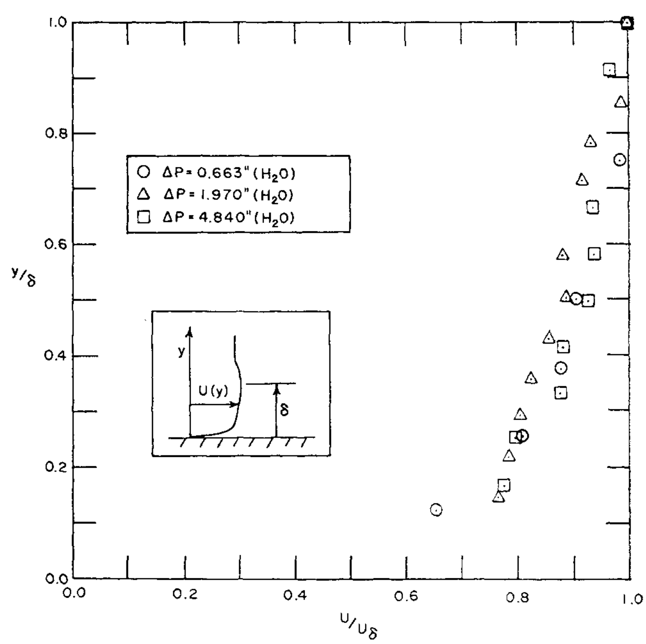

Fig. 5 Dimensionless boundary-layer profile.

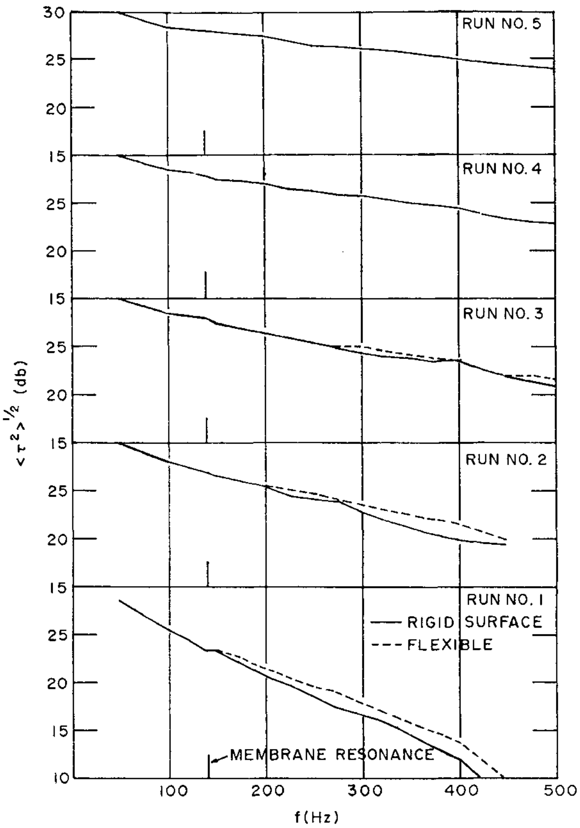

Fig. 6 Time-dependent shear-stress spectrum.

velocity increases. This peaking of the amplitude response has been observed by others[4–6] and is referred to as the coincidence effect since the peak value occurs when the convection velocity in the boundary-layer and the elastic wave velocity in the membrane are equal. The damping in the membrane as presented in Fig. 8 is shown to be velocity dependent, a phenomenon also observed in Refs. 4, 5, and 7.

Discussion and Conclusions

A definite influence of a membrane's motions on the time-dependent shear stress is observed. The influence is the strongest at the lowest flow velocity and not observed at the highest velocities. The reason for this can be seen in the

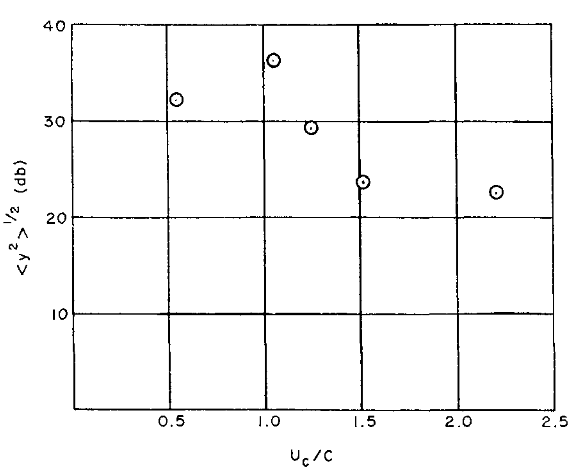

Fig. 7 Velocity-dependent membrane amplitude response.

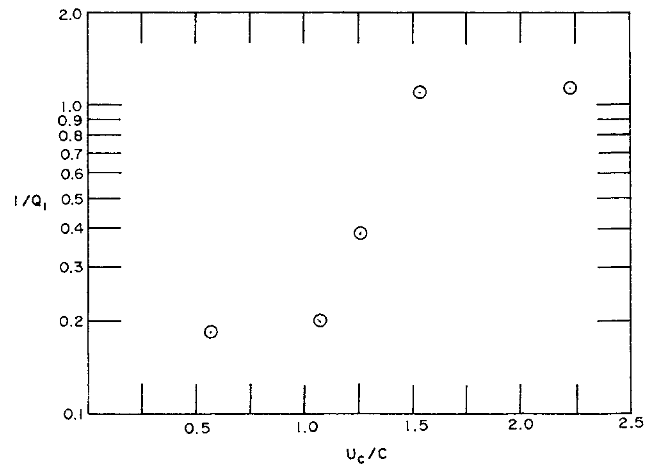

Fig. 8 Velocity-dependent damping.

damping data in Fig. 8. As the velocity increases, the damping increases by nearly an order of magnitude over the velocity range. This causes a significant reduction in the amplitude response of the membrane as shown in Fig. 7, i.e., 14 *db* between the coincidence amplitude and the high velocity amplitude.

At the lower velocities, the surface motions cause a shift in the energy spectrum of the turbulence above the membrane's resonance frequency. The higher wave number eddies would appear to become somewhat displaced from their normal position in the boundary layer. The resulting shift in frequency may then be related to a change in the convection velocity of the high wave number eddies as detected by the flush mounted sensor.

The damping variation with flow velocity is believed to be radiation damping. The magnitude of the contribution of the reflections from the tunnel walls is uncertain. Results of boundary-layer induced vibration studies on towed bodies would help shed light on the nature of the damping.

Measurements of the time-averaged flow properties, such as velocity profile and surface shear stress, showed no apparent effects from the membrane's motions. Thus, at least for small surface motions, the only properties affected by a compliant surface are those which are time-dependent.

References

[1] Corcos, G. M., "Resolution of Pressure in Turbulence," *Journal of the Acoustical Society of America*, Vol. 35, No. 2, February 1963, pp. 192–199.

[2] Willmarth, W. W. and Yang, C. S., "Wall-Pressure Fluctuations Beneath Turbulent Boundary-Layers on a Flat Plate and on a Cylinder," *Journal of Fluid Mechanics*, Vol. 41, Pt. 1, March 1970, pp. 47–80.

[3] Maestrello, L. and Linden, T. L. J., "Study of the Response of a Vibrating Plate Immersed in a Fluid," *8th Symposium on Naval Hydromechanics*, Aug. 1970, California Institute of Technology, Pasadena, Calif.

[4] McCormick, M. E. and Ripley, T. C., "On Turbulence-Induced Vibrations of a Thin Ribbon with Velocity-Dependent Damping," *Journal of Applied Mechanics*, Vol. 37, Ser. E, No. 4, 1970, pp. 1172–1176.

[5] Lyon, R. H., "Response of Strings to Random Noise Fields," *Journal of the Acoustical Society of America*, Vol. 28, No. 3, March 1956, pp. 391–398.

[6] Tack, D. H. and Lambert, R. F., "Response of Bars and Plates to Boundary-Layer Turbulence," *Journal of Aerospace Sciences*, Vol. 29, No. 3, March 1962, pp. 311–322.

[7] Wambsganss, M. W. and Zaleski, P. L., "Flow Velocity Dependence of Damping in Parallel-Flow-Induced Vibration," *Transactions of the American Nuclear Society*, Vol. 12, 1969, pp. 839–840.